Piecewise-Defined Functions Homework

For problems 1-3, evaluate each piecewise function at the given values of the independent variable.

1. $f(x)=\begin{cases}6x-1 & \text{if } x<0\\ 7x+3 & \text{if } x\geq 0\end{cases}$ a. $f(-3)$ b. $f(0)$ c. $f(4)$

2. $f(x)=\begin{cases}\dfrac{x^2-9}{x+2} & \text{if } x\leq -1\\ 6 & \text{if } x>-1\end{cases}$ a. $f(-3)$ b. $f(1)$

3. $f(x)=\begin{cases}2+x & \text{if } x<-4\\ -x & \text{if } -4\leq x\leq 2\\ \dfrac{1}{3}x & \text{if } x>2\end{cases}$ a. $f(2)$ b. $f(3)$

4. When a diabetic takes long-acting insulin, the insulin reaches its peak effect on the blood sugar level in about three hours. This effect remains fairly constant for 5 hours, then declines, and is very low until the next injection. In a typical patient, the level of insulin might be modeled by the following function.

$$f(t)=\begin{cases}40t+100 & \text{if } 0\leq t\leq 3\\ 220 & \text{if } 3<t\leq 8\\ -80t+860 & \text{if } 8<t\leq 10\\ 60 & \text{if } 10<t\leq 24\end{cases}$$

Here, $f(t)$ represents the blood sugar level at time t hours after the time of the injection. If a patient takes insulin at 6 am, find the blood sugar level at each of the following times.

a. 7 am b. 11 am c. 3 pm d. 5 pm

For problems 5-14, graph each piecewise function.

5. $f(x)=\begin{cases}x+3 & \text{if } x<-1\\ 2x-1 & \text{if } x\geq -1\end{cases}$

6. $f(x)=\begin{cases}x-1 & \text{if } x\leq 3\\ 2 & \text{if } x>3\end{cases}$

7. $f(x)=\begin{cases}-1 & \text{if } x<0\\ x-3 & \text{if } x\geq 0\end{cases}$

8. $f(x)=\begin{cases}4-x & \text{if } x\leq 2\\ 3x-6 & \text{if } x>2\end{cases}$

9. $f(x)=\begin{cases} -2x & \text{if } x<-1 \\ 3x-1 & \text{if } -1\le x\le 2 \\ -\frac{1}{2}x & \text{if } x>2 \end{cases}$

10. $f(x)=\begin{cases} 2+x & \text{if } x<-2 \\ -x & \text{if } -2\le x\le 1 \\ 0 & \text{if } x>1 \end{cases}$

11. $f(x)=\begin{cases} x & \text{if } x\le -2 \\ 2x & \text{if } x>-2 \end{cases}$

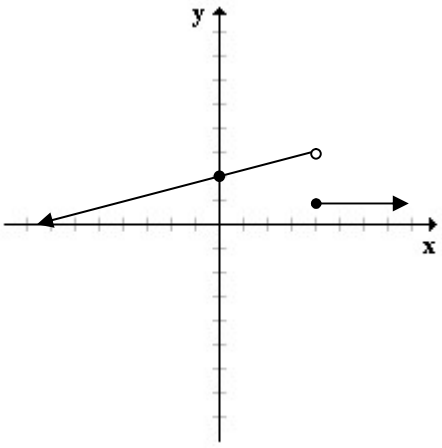
12. $f(x)=\begin{cases} -2 & \text{if } x<0 \\ 2 & \text{if } x>0 \end{cases}$

13. $f(x)=\begin{cases} 2x+1 & \text{if } x<-1 \\ 2x+2 & \text{if } x\ge -1 \end{cases}$

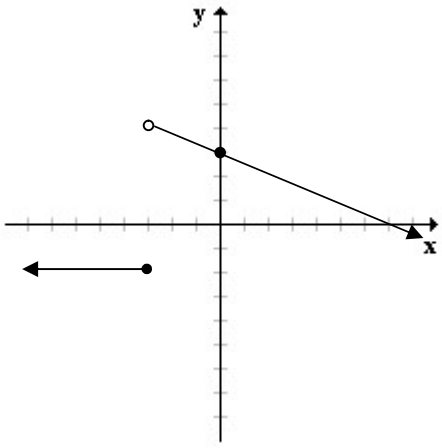
14. $f(x)=\begin{cases} 2x-1 & \text{if } x\le 0 \\ 2-x & \text{if } 0<x<3 \\ x+1 & \text{if } x\ge 3 \end{cases}$

For problems 15-17, give the piecewise function that each graph represents.

15.



16.



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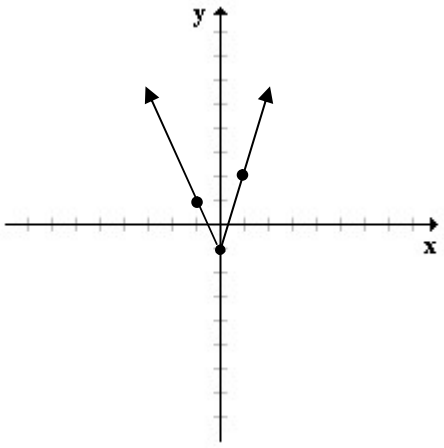


For problems 18-21, write a piecewise function that describes each situation.

18. For a cellular phone billing plan, $50 per month buys 400 minutes or less. Additional time costs $0.30 per minute. Let the monthly cost C(x) be a function of the time x.

19. For a cellular phone billing plan, $60 per month buys 450 minutes or less. Additional time costs $0.35 per minute. Let the monthly cost C(x) be a function of the time x.

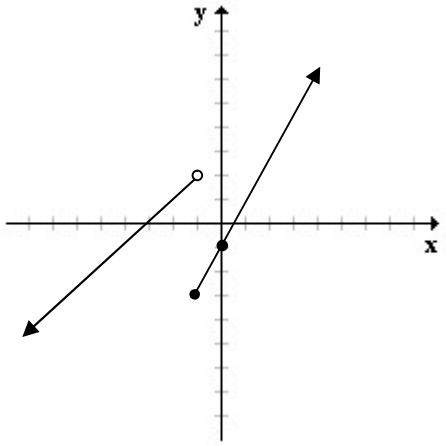
20. Income tax is 5% on the first $50,000 of income or less, and 8% on any income in excess of $50,000. Let the tax T(x) be a function of the income x.

21. In Missouri, income tax is 3.5% on the first $9,000 of income or less, and 6% on any income in excess of $9,000. Let the tax T(x) be a function of the income x.

Odd Solutions – Piecewise Functions

1a. $f(-3) = -19$ 1b. $f(0) = 3$ 1c. $f(4) = 31$ 3a. $f(2) = -2$ 3b. $f(3) = 1$

5.



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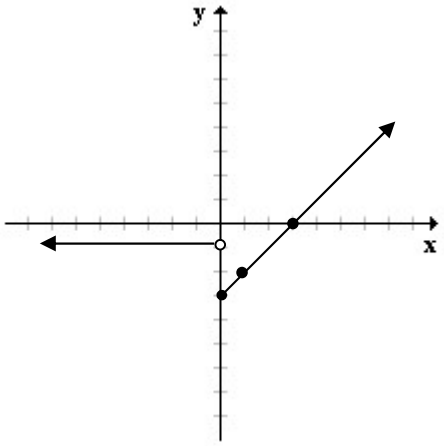

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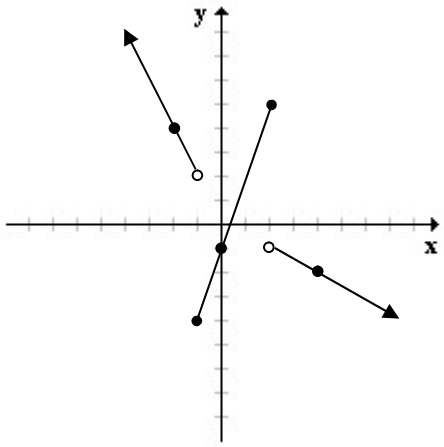

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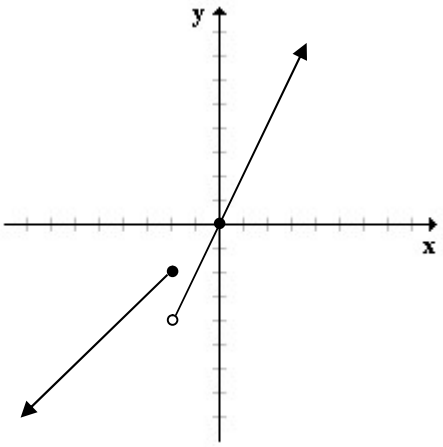

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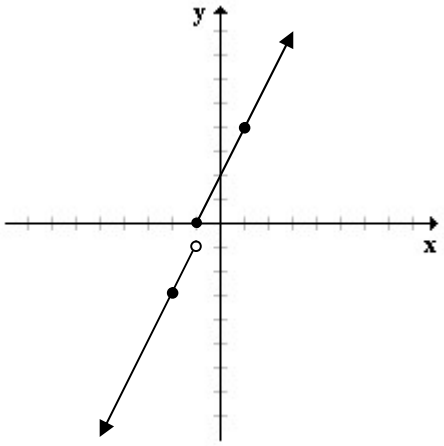

15. $f(x) = \begin{cases} \frac{1}{4}x + 2 & \text{if } x < 4 \\ 1 & \text{if } x \geq 4 \end{cases}$

17. $f(x) = \begin{cases} -2x - 1 & \text{if } x \leq 0 \\ 3x - 1 & \text{if } x > 0 \end{cases}$ or $f(x) = \begin{cases} -2x - 1 & \text{if } x < 0 \\ 3x - 1 & \text{if } x \geq 0 \end{cases}$

19. $C(x) = \begin{cases} 60 & \text{if } 0 \leq x \leq 450 \\ 60 + 0.35(x - 450) & \text{if } x > 450 \end{cases}$ or $C(x) = \begin{cases} 60 & \text{if } 0 \leq x \leq 450 \\ 0.35x - 97.5 & \text{if } x > 450 \end{cases}$

21. $T(x) = \begin{cases} 0.035x & \text{if } 0 \leq x \leq 9000 \\ 0.035(9000) + 0.06(x - 9000) & \text{if } x > 9000 \end{cases}$ or $T(x) = \begin{cases} 0.035x & \text{if } 0 \leq x \leq 9000 \\ 0.06x - 225 & \text{if } x > 9000 \end{cases}$